

Exact Green Function for Neutral Pauli-Dirac Particle with Anomalous Magnetic Momentum in Linear Magnetic Field

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Abstract

We consider Pauli–Dirac fermion submitted to an inhomogeneous magnetic field. It is showed that the propagator of the neutral Dirac particle with an anomalous magnetic moment in an external linear magnetic field is the causal Green function $S^c(x_b, x_a)$ of the Pauli–Dirac equation. The corresponding Green function is calculated via path integral method in global projection, giving rise to the exact eigenspinors expressions. The neutral particle creation probability corresponding to our system is analyzed, which is obtained as function of the introduced field B' and the additional spin magnetic moment μ .

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1 Introduction

In relativistic quantum mechanics, it well-know that the Dirac equation does not correctly describe particles of spin $1/2$. This is due to the corrections imported by the quantum electrodynamics, such as vacuum polarization or pair creations, to the intrinsic magnetic momentum. With this respect, the first attempt was due to Pauli who suggested to add a term describing the anomalous of the magnetic momentum of particle in the Dirac equation [1], which is later on called Pauli–Dirac equation. With this extended equation the exact or approximate solutions of the eigenvalue problem are very much needed. However, it is rare to find such solutions except for interaction with electromagnetic fields: constant uniform magnetic field, electromagnetic plane wave and more complicated ones [2]. In particular, we recall that the Dirac equation for charged and neutral fermions with anomalous magnetic moments was solved in a uniform magnetic field [3]. The expressions of relativistic wave functions and energy spectra were explicitly obtained. It was showed that, in the nonrelativistic limit the wave functions and energy spectra of charged fermions agreed with the known solutions of the Schrödinger equation. The dependence of the relativistic wave functions on the magnetic quantum number was discussed in more detail and the obtained results were compared with the literature [4,5].

In theoretical physics there is a formalism that became a powerful mathematical tool, this is the path integral introduced by Feynman years ago. Even it is successfully applied in the field theory and quantum mechanics but faced serious problems with the relativistic quantum mechanics. Such problems are related to the covariance that requires treating the space and time in equally as well as taking into account of the dynamics of spin as an additional degree of freedom. In 1951, the first problem was solved by Schwinger [6], who used an approach based on the proper time parametrization and allowed him to explicitly determine the corrected magnetic momentum. However, the second problem is much more complicated because of the nature of the spin momentum. More precisely, there is an interplay between two sets such that the spin is a discrete object (quantum mechanics) and the path integral is based on the trajectory notion (classical mechanics). To overcome such difficulties and after several attempts to construct the Green function corresponding to the relativistic particles using the path integral mechanism, Fradkin and Gitman were succeeded in establishing an interesting mathematical formalism [7].

Motivated by different works and specifically reference [3], we study the problem of the relativistic Pauli-Dirac neutral particle submitted to a linear magnetic field by adopting the path integral mechanism. In the first step, we show that the global projection gives rise the Green function of the inverse Pauli–Dirac operator. This will be used together with some algebra to explicitly determine the corresponding eigenspinors as well as analyze their asymptotic behaviors. Subsequently, we determine the probability of pair creation \mathcal{W} in vacuum in 2+1 dimensions corresponding to our system. This can be done by making use the effective action together with the charge conjugation matrix to end up with \mathcal{W} in terms of the additional spin magnetic momenta and magnetic field.

The paper is organized as follows. In section 2, we show that how one can use the global projection for $(2 + 1)$ -dimensional Pauli-Dirac equation in linear magnetic field. In section 3, we give the theoretical formulation of the problem where different changes are introduced to simplify the process for obtaining the solutions of eigenspinors. More precisely, we will use the causal Green function as well as different techniques to solve our problem. We determine the eigenspinors in terms of different

physical parameters and quantum numbers in section 4. In section 5, we calculate the probability of pair creation in terms of different parameters. We conclude our findings in the final section.

2 Global projection

We start by recalling that the Clifford algebra with three generating elements has two inequivalent two-dimensional irreducible representations of gamma matrices. These will serve as a guide to describe the behavior of a system of relativistic charged particle in constant magnetic field with a given value of the helicity. In doing so, we show that the global projection giving rise the Green function of the inverse Pauli–Dirac operator O_-^{-1}

$$S^c = \left(\gamma_\varsigma^\mu p_\mu - m - \frac{\mu}{2} \sigma^{\mu\nu} F_{\mu\nu} \right)^{-1} = O_-^{-1} = O_+ (O_- O_+)^{-1} \quad (1)$$

where the operators O_\pm read as

$$O_\pm = \gamma_\varsigma^\mu p_\mu - \frac{\mu}{2} \sigma^{\mu\nu} F_{\mu\nu} \pm m \quad (2)$$

while μ describes the additional spin magnetic moment. The matrices γ_ς^μ are defined through the following relations

$$[\gamma_\varsigma^\mu, \gamma_\varsigma^\nu]_+ = 2\eta^{\mu\nu}, \quad [\gamma_\varsigma^\mu, \gamma_\varsigma^\nu]_- = -2i\sigma^{\mu\nu} \quad (3)$$

generating the Clifford algebra, which is giving rise to the algebra of different representations for Dirac gamma matrices, labeled by the subscript $\varsigma = \pm 1$. particularly in (2+1)-dimensions the metric is $\eta^{\mu\nu} = \text{diag}(1, -1, -1)$ and the matrices are mapped as

$$\gamma_\varsigma^\mu = \frac{i\varsigma}{2} \epsilon^{\mu\nu\lambda} \gamma_{\varsigma,\nu} \gamma_{\varsigma,\lambda}, \quad \mu, \nu = 0, 1, 2 \quad (4)$$

or equivalently

$$\gamma_\varsigma^0 = i\varsigma \gamma_\varsigma^1 \gamma_\varsigma^2 = \sigma_3, \quad \gamma_\varsigma^1 = -i\varsigma \gamma_\varsigma^0 \gamma_\varsigma^2 = i\sigma_2, \quad \gamma_\varsigma^2 = -i\varsigma \gamma_\varsigma^0 \gamma_\varsigma^1 = -i\sigma_1. \quad (5)$$

Now from (1), one can see that the Green function S^c satisfies the equation

$$\left(\gamma_\varsigma^\mu p_\mu - m - \frac{\mu}{2} \sigma^{\mu\nu} F_{\mu\nu} \right) S^c = -\mathbb{I}_{3 \otimes 3}. \quad (6)$$

Note that, formally $S^c(\mathbf{x}_b, \mathbf{x}_a)$ is the matrix element in the coordinate space and therefore it can be written as

$$S^c(\mathbf{x}_b, \mathbf{x}_a) = \langle \mathbf{x}_b | S^c | \mathbf{x}_a \rangle. \quad (7)$$

It is clearly seen that, if we multiply both sides of (6) by the states $\langle \mathbf{x}_b |$ and $| \mathbf{x}_a \rangle$, we end up with the following equation for $S^c(\mathbf{x}_b, \mathbf{x}_a)$

$$\left(\gamma_\varsigma^\mu (i\partial_{b\mu}) - m - i\frac{\mu}{2} \gamma_\varsigma^\mu \gamma_\varsigma^\nu F_{\mu\nu}(\mathbf{x}_b) S^c(\mathbf{x}_b, \mathbf{x}_a) \right) = -\delta^3(\mathbf{x}_b - \mathbf{x}_a) \quad (8)$$

which is nothing but the Pauli–Dirac equation that will play a crucial role in our analysis and allows to deal with our task.

Having the expressions for the operators O_+ and O_- , we can define the Green operator for global projection by

$$S_g^c = (O_- O_+)^{-1} = (O_+ O_-)^{-1} \quad (9)$$

where the label g stands for global. One can easily show that the matrix element of S_g^c verifies the quadratic Dirac equation

$$O_- O_+ S_g^c(\mathbf{x}_b, \mathbf{x}_a) = O_+ O_- S_g^c(\mathbf{x}_b, \mathbf{x}_a) = -\delta^3(\mathbf{x}_b - \mathbf{x}_a) \quad (10)$$

Now we can establish a relation between the matrix elements $S^c(\mathbf{x}_b, \mathbf{x}_a)$ and $S_g^c(\mathbf{x}_b, \mathbf{x}_a)$ of both projections. Indeed, using (2), (8) and (10) to end up with

$$S^c(\mathbf{x}_b, \mathbf{x}_a) = \left(\gamma^\mu (i\partial_{b\mu}) + m - i\frac{\mu}{2} \gamma_\zeta^\mu \gamma_\zeta^\nu F_{\mu\nu}(\mathbf{x}_b) \right) S_g^c(\mathbf{x}_b, \mathbf{x}_a). \quad (11)$$

In the next, we show how the above mathematical tools can be used to deal with different issues those concern the eigenspinors of our system as well as the probability of pair creation in vacuum.

3 Green function

To achieve our goals, let us fix our system by considering the interaction of a neutral fermion with a linear magnetic field B . It is convenient for our task to choose an interaction described by the following quadri-potential

$$A_0 = 0, \quad A_x = 0, \quad A_y = Bx + \frac{1}{2}B'x^2 \quad (12)$$

where the field B' is introduced to generalize the constant magnetic field cases and more importantly will allow to deal with the pair creation. One can use this gauge to show that the operators (2) take the forms

$$O_\pm = \sigma_3 \left(\hat{p}_0 - \varsigma\mu \frac{B + B'x}{2} \right) - i\sigma_2 \hat{p}_x + i\varsigma\sigma_1 \hat{p}_y \pm m. \quad (13)$$

These allow us to define $S_g^c(\mathbf{x}_b, \mathbf{x}_a)$ as

$$\begin{aligned} S_g^c(\mathbf{x}_b, \mathbf{x}_a) &= \langle \mathbf{x}_b | (O_+ O_-)^{-1} | \mathbf{x}_a \rangle \\ &= -\frac{i}{2} \int_0^\infty d\lambda \langle \mathbf{x}_b | \exp \left(i\frac{\lambda}{2} (\mathcal{H} + i\varepsilon) \right) | \mathbf{x}_a \rangle \end{aligned} \quad (14)$$

where $i\varepsilon$ is the Feynman prescription and the involved Hamiltonian $\mathcal{H} = O_+ O_- = O_- O_+$ is given by

$$\mathcal{H} = \left(\hat{p}_0 - \varsigma\mu \frac{B + B'x}{2} \right)^2 - \hat{p}_x^2 - \hat{p}_y^2 - m^2 + i\varsigma \frac{\mu B'}{2} \sigma_1 \quad (15)$$

which can be written under the unitary transformation $e^{+i\frac{\pi}{4}\sigma_2}$ as

$$\mathcal{H} = e^{-i\frac{\pi}{4}\sigma_2} \left[\left(\hat{p}_0 - \varsigma\mu \frac{B + B'x}{2} \right)^2 - \hat{p}_x^2 - \hat{p}_y^2 - m^2 + i\varsigma \frac{\mu B'}{2} \sigma_3 \right] e^{+i\frac{\pi}{4}\sigma_2}. \quad (16)$$

In the path integral representation, $S_g^c(\mathbf{x}_b, \mathbf{x}_a)$ takes the form

$$\begin{aligned} S_g^c(\mathbf{x}_b, \mathbf{x}_a) &= -\frac{i}{2} \int_0^\infty d\lambda \int Dt Dx Dy \int Dp_0 Dp_x Dp_y e^{-i\frac{\pi}{4}\sigma_2} e^{-\varsigma\frac{\mu B'}{4}\sigma_3} e^{+i\frac{\pi}{4}\sigma_2} \\ &\quad \exp \left(i \int_0^1 \left[p_0 \dot{t} - p_x \dot{x} - p_y \dot{y} + \frac{\lambda}{2} \left(\left(p_0 - \varsigma\mu \frac{B + B'x}{2} \right)^2 - p_x^2 - p_y^2 - m^2 \right) \right] d\tau \right) \end{aligned} \quad (17)$$

where $\mathbf{x} = (t, x, y)$ satisfying the boundary conditions

$$\mathbf{x}(0) = \mathbf{x}_a, \quad \mathbf{x}(1) = \mathbf{x}_b. \quad (18)$$

Integrating over the paths t and y , one can see that the momenta become constants, i.e. $p_0 = \text{const}$ and $p_y = \text{const}$. Again integrating once more over p_x , to rewrite $S_g^c(\mathbf{x}_b, \mathbf{x}_a)$ as

$$S_g^c(\mathbf{x}_b, \mathbf{x}_a) = -\frac{i}{2} \int_0^\infty d\lambda \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} e^{-ip_0(t_b-t_a)+ip_y(y_b-y_a)-\frac{i\lambda}{2}(p_y^2+m^2)} e^{-i\frac{\pi}{4}\sigma_2} e^{-\lambda\varsigma\frac{\mu B'}{4}\sigma_3} e^{i\frac{\pi}{4}\sigma_2} K^{os}(x_a, x_b; \lambda) \quad (19)$$

where K^{os} is the propagator related to the paths $x(\tau)$ and expresses the motion of a particle submitted to an harmonic oscillator with an imaginary frequency $\omega = i\frac{\mu B'}{2}$ and mass $m = 1$, which is

$$K^{os}(x_a, x_b; \lambda) = \int Dx e^{i \int_0^1 \left(\frac{\dot{x}^2}{2\lambda} + \lambda \frac{\mu^2 B'^2}{8} \left(x - \frac{2\varsigma p_0 - \mu B}{\mu B'} \right)^2 \right) d\tau}. \quad (20)$$

The integral over the paths $x(\tau)$ is well-known and equal to

$$K^{os}(x_a, x_b; \lambda) = \left(\frac{\frac{\mu B'}{2}}{2i\pi \sinh\left(\frac{\lambda \mu B'}{2}\right)} \right)^{1/2} \exp\left(i \frac{\left(z_b^2 + z_a^2 \right) \cosh\left(\frac{\lambda \mu B'}{2}\right) - 2z_b z_a}{4 \sinh\left(\frac{\lambda \mu B'}{2}\right)} \right) \quad (21)$$

where $z = \sqrt{\mu B'} \left(x - \frac{2\varsigma p_0 - \mu B}{\mu B'} \right)$. The propagator K^{os} can be expressed as [8]

$$K^{os} = \frac{1}{\sqrt{1+\xi^2}} \exp\left[\frac{1}{4} \frac{1-\xi^2}{1+\xi^2} (\alpha^2 + \beta^2) + i\alpha\beta \frac{\xi}{1+\xi^2} \right] \quad (22)$$

or in terms of the parabolic cylindrical function $D_\nu(z)$

$$K^{os} = \frac{\left(\frac{\pi}{2}\right)^{1/2}}{2\pi i} \int_{\iota-i\infty}^{\iota+i\infty} \frac{\xi^{-(\nu+1)}}{\sin(-\pi\nu)} \sum_{\epsilon=\pm 1} D_\nu\left(\epsilon e^{-i\frac{\pi}{4}}\alpha\right) D_{-\nu-1}\left(\epsilon e^{i\frac{\pi}{4}}\beta\right) d\nu \quad (23)$$

and the used notations are $-1 < \iota < 0$, $\xi = e^{i\frac{\pi}{2}} e^{\frac{\lambda \mu B'}{2}}$ ($|\arg \xi| < \frac{\pi}{2}$), $\alpha = z_b$, $\beta = z_a$. After inserting (23) into (19) we obtain

$$S_g^c(\mathbf{x}_b, \mathbf{x}_a) = -\frac{1}{4\pi} \left(\frac{\mu B'}{4} \right)^{1/2} \int_0^\infty d\lambda \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} e^{-ip_0(t_b-t_a)+ip_y(y_b-y_a)-\frac{i\lambda}{2}(p_y^2+m^2)} \int_{\iota-i\infty}^{\iota+i\infty} \frac{\xi^{-(\nu+1/2)}}{\sin(-\pi\nu)} e^{-i\frac{\pi}{4}\sigma_2} e^{-\varsigma\frac{\mu B'}{4}\sigma_3} e^{i\frac{\pi}{4}\sigma_2} \sum_{\epsilon=\pm 1} D_\nu\left(\epsilon e^{-i\frac{\pi}{4}}z_b\right) D_{-\nu-1}\left(\epsilon e^{i\frac{\pi}{4}}z_a\right) d\nu. \quad (24)$$

To go further in developing (24), let us introduce the spin operator σ_3 and insert the identity $\sum_{s=\pm 1} \chi_s \chi_s^+ = \mathbb{I}_{2\otimes 2}$. We can easily check the relations

$$\sigma_3 \chi_s = s \chi_s, \quad \sigma_1 \chi_s = \chi_{-s}, \quad \sigma_2 \chi_s = i s \chi_{-s} \quad (25)$$

for the vectors

$$\chi_{+1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_{-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (26)$$

which can be implemented in (24) to obtain the causal Green function $S_g^c(\mathbf{x}_b, \mathbf{x}_a)$

$$S_g^c = -\frac{(\mu B')^{1/2}}{8\pi} \int_0^\infty d\lambda \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} e^{-ip_0(t_b-t_a)+ip_y(y_b-y_a)-\frac{i\lambda}{2}(p_y^2+m^2)} \sum_{s=\pm 1} e^{-\varsigma \frac{\mu B'}{4}s} e^{-i\frac{\pi}{4}\sigma_2} \chi_s \chi_s^+ e^{+i\frac{\pi}{4}\sigma_2} \int_{\iota-i\infty}^{\iota+i\infty} \frac{\xi^{-(\nu+1/2)}}{\sin(-\pi\nu)} e^{-i\frac{\pi}{4}\sigma_2} \sum_{\epsilon=\pm 1} D_\nu \left(\epsilon e^{-i\frac{\pi}{4}} z_b \right) D_{-\nu-1} \left(\epsilon e^{i\frac{\pi}{4}} z_a \right) d\nu. \quad (27)$$

Using the identity

$$\sum_{s=\pm 1} f_s \sum_{\epsilon=\pm 1} g_\epsilon = \sum_{s=\pm 1} f_s (g_s + g_{-s}) \quad (28)$$

to write $S_g^c(\mathbf{x}_b, \mathbf{x}_a)$ as

$$S_g^c = -\frac{(\mu B')^{1/2}}{8\pi} \int_0^\infty d\lambda \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} e^{-ip_0(t_b-t_a)+ip_y(y_b-y_a)-\frac{i\lambda}{2}(p_y^2+m^2)} \sum_{s=\pm 1} e^{-\varsigma \frac{\mu B'}{4}s} e^{-i\frac{\pi}{4}\sigma_2} \chi_s \chi_s^+ e^{+i\frac{\pi}{4}\sigma_2} \int_{\iota-i\infty}^{\iota+i\infty} \frac{\xi^{-(\nu+1/2)}}{\sin(-\pi\nu)} \left[D_\nu \left(s e^{-i\frac{\pi}{4}} z_b \right) D_{-\nu-1} \left(s e^{i\frac{\pi}{4}} z_a \right) + D_\nu \left(-s e^{-i\frac{\pi}{4}} z_b \right) D_{-\nu-1} \left(-s e^{i\frac{\pi}{4}} z_a \right) \right] d\nu. \quad (29)$$

Once we integrate over λ , we obtain $\left[\frac{i}{2} (p_y^2 + m^2 - i\mu B' (\nu + \frac{1+\varsigma s}{2})) \right]^{-1}$, which appears at the denominator and out from the following pole

$$\nu = -\frac{1+\varsigma s}{2} - i\rho \quad (30)$$

where we have introduced the quantity $\rho = \frac{p_y^2+m^2}{\mu B'}$. We finally obtain

$$S_g^c = \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} e^{-ip_0(t_b-t_a)+ip_y(y_b-y_a)} \sum_{s=\pm 1} \frac{e^{i\pi \frac{\varsigma s}{4} - \frac{\pi}{2} \rho \varsigma s}}{2(\mu B')^{1/2} \sinh(\pi\rho)} e^{-i\frac{\pi}{4}\sigma_2} \chi_s \chi_s^+ e^{+i\frac{\pi}{4}\sigma_2} \left[D_{-\frac{1+\varsigma s}{2}-i\rho} \left(s e^{-i\frac{\pi}{4}} z_b \right) D_{-\frac{1-\varsigma s}{2}+i\rho} \left(s e^{i\frac{\pi}{4}} z_a \right) + D_{-\frac{1+\varsigma s}{2}-i\rho} \left(-s e^{-i\frac{\pi}{4}} z_b \right) D_{-\frac{1-\varsigma s}{2}+i\rho} \left(-s e^{i\frac{\pi}{4}} z_a \right) \right]. \quad (31)$$

For more information about the parabolic cylindrical functions of the form $D_{-\frac{1}{2}+i\alpha} \left(\pm e^{-i\frac{\pi}{4}} z \right)$ and their properties, we refer to [12].

4 Eigenspinors

At this stage, we show how the obtained results above can be used to explicitly determine the eigenspinors corresponding to our system. To this end, we map (31) into the causal Green function (11) to end up with

$$S^c = \left[\left(i \frac{\partial}{\partial t_b} - \varsigma \mu \frac{B+B'x_b}{2} \right) \sigma_3 - i\sigma_2 \left(-i \frac{\partial}{\partial x_b} \right) + i\varsigma \sigma_1 \left(-i \frac{\partial}{\partial y_b} \right) + m \right] \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} e^{-ip_0(t_b-t_a)+ip_y(y_b-y_a)} \sum_{s=\pm 1} \frac{e^{i\pi \frac{\varsigma s}{4} - \frac{\pi}{2} \rho \varsigma s}}{2(\mu B')^{1/2} \sinh(\pi\rho)} e^{-i\frac{\pi}{4}\sigma_2} \chi_s \chi_s^+ e^{+i\frac{\pi}{4}\sigma_2} \left[D_{-\frac{1+\varsigma s}{2}-i\rho} \left(s e^{-i\frac{\pi}{4}} z_b \right) D_{-\frac{1-\varsigma s}{2}+i\rho} \left(s e^{i\frac{\pi}{4}} z_a \right) + D_{-\frac{1+\varsigma s}{2}-i\rho} \left(-s e^{-i\frac{\pi}{4}} z_b \right) D_{-\frac{1-\varsigma s}{2}+i\rho} \left(-s e^{i\frac{\pi}{4}} z_a \right) \right] \quad (32)$$

which gives the Green function relative to our particle as

$$S^c(\mathbf{x}_b, \mathbf{x}_a) = \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} e^{-ip_0(t_b-t_a)+ip_y(y_b-y_a)} \sum_{s=\pm 1} \frac{e^{i\pi \frac{\varsigma s}{4} - \frac{\pi}{2} \rho \varsigma}}{2 \sinh(\pi \rho)} e^{-i\frac{\pi}{4} \sigma_2} \quad (33)$$

$$\left[\left(-is\sqrt{\mu B'} \left(\frac{\partial}{\partial z_b} + i\frac{\varsigma s}{2} z_b \right) \chi_{-s} \chi_s^+ + (i\varsigma s p_y + m) \chi_s \chi_s^+ \right) \right] e^{+i\frac{\pi}{4} \sigma_2} \left[D_{-\frac{1+\varsigma s}{2}-i\rho} \left(se^{-i\frac{\pi}{4}} z_b \right) \right. \\ \left. D_{-\frac{1-\varsigma s}{2}+i\rho} \left(se^{i\frac{\pi}{4}} z_a \right) + D_{-\frac{1+\varsigma s}{2}-i\rho} \left(-se^{-i\frac{\pi}{4}} z_b \right) D_{-\frac{1-\varsigma s}{2}+i\rho} \left(-se^{i\frac{\pi}{4}} z_a \right) \right].$$

For later use we introduce the following recurrence and derivative relations between the parabolic cylindrical functions [10]

$$z D_\nu(z) = D_{\nu+1}(z) + \nu D_{\nu-1}(z) \quad (34)$$

$$\frac{\partial}{\partial z} D_\nu(z) = \frac{1}{2} \nu D_{\nu-1}(z) - \frac{1}{2} D_{\nu+1}(z). \quad (35)$$

After verification for all values $\varsigma s = \pm 1$, we obtain the interesting property

$$\left(\frac{d}{dz} - \frac{\varsigma s}{2} z \right) D_{-\frac{1+\varsigma s}{2}-i\rho}(z) = -\frac{(1-\varsigma s)\rho+(1+\varsigma s)}{2} D_{-\frac{1-\varsigma s}{2}-i\rho}(z) \quad (36)$$

which implies the relation

$$\left(\frac{d}{dz_b} - \frac{\varsigma s}{2} z_b \right) D_{-\frac{1+\varsigma s}{2}-i\rho} \left(se^{-i\frac{\pi}{4}} z_b \right) = -se^{-i\frac{\pi}{4} \varsigma s} \frac{(1-\varsigma s)\rho+(1+\varsigma s)}{2} D_{-\frac{1-\varsigma s}{2}-i\rho} \left(se^{-i\frac{\pi}{4}} z_b \right). \quad (37)$$

Using all to write $S^c(\mathbf{x}_b, \mathbf{x}_a)$ as

$$S^c(\mathbf{x}_b, \mathbf{x}_a) = \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} e^{-ip_0(t_b-t_a)+ip_y(y_b-y_a)} \sum_{s=\pm 1} \frac{e^{-\frac{\pi}{2} \rho \varsigma}}{2 \sinh(\pi \rho)} \\ e^{-i\frac{\pi}{4} \sigma_2} \left\{ is \frac{(1-\varsigma s)\rho+(1+\varsigma s)}{2} D_{-\frac{1-\varsigma s}{2}-i\rho} \left(se^{-i\frac{\pi}{4}} z_b \right) D_{-\frac{1-\varsigma s}{2}+i\rho} \left(se^{i\frac{\pi}{4}} z_a \right) \chi_{-s} \chi_s^+ \right. \\ \left. - is \frac{(1-\varsigma s)\rho+(1+\varsigma s)}{2} D_{-\frac{1-\varsigma s}{2}-i\rho} \left(-se^{-i\frac{\pi}{4}} z_b \right) D_{-\frac{1-\varsigma s}{2}+i\rho} \left(-se^{i\frac{\pi}{4}} z_a \right) \chi_{-s} \chi_s^+ \right. \\ \left. + se^{i\frac{\pi}{4} \varsigma s} \frac{i\varsigma s p_y + m}{\sqrt{\mu B'}} D_{-\frac{1+\varsigma s}{2}-i\rho} \left(se^{-i\frac{\pi}{4}} z_b \right) D_{-\frac{1-\varsigma s}{2}+i\rho} \left(se^{i\frac{\pi}{4}} z_a \right) \chi_s \chi_s^+ \right. \\ \left. + se^{i\frac{\pi}{4} \varsigma s} \frac{i\varsigma s p_y + m}{\sqrt{\mu B'}} D_{-\frac{1+\varsigma s}{2}-i\rho} \left(-se^{-i\frac{\pi}{4}} z_b \right) D_{-\frac{1-\varsigma s}{2}+i\rho} \left(-se^{i\frac{\pi}{4}} z_a \right) \chi_s \chi_s^+ \right\} e^{+i\frac{\pi}{4} \sigma_2}. \quad (38)$$

In developing further the above relation, let us introduce the mapping $s \rightarrow -s$ in (38) but only for the terms $D_{-\frac{1+\varsigma s}{2}-i\rho} \left(-se^{-i\frac{\pi}{4}} z_b \right) D_{-\frac{1-\varsigma s}{2}+i\rho} \left(-se^{i\frac{\pi}{4}} z_a \right)$, to get the form

$$S^c(\mathbf{x}_b, \mathbf{x}_a) = \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} e^{-ip_0(t_b-t_a)+ip_y(y_b-y_a)} \sum_{s=\pm 1} \frac{e^{-\frac{\pi}{2} \rho \varsigma}}{2 \sinh(\pi \rho)} \\ e^{-i\frac{\pi}{4} \sigma_2} is \left\{ \frac{(1-\varsigma s)\rho+(1+\varsigma s)}{2} D_{-\frac{1-\varsigma s}{2}-i\rho}(sZ_b) D_{-\frac{1-\varsigma s}{2}+i\rho}(sZ_a) \chi_{-s} \chi_s^+ \right. \\ \left. + \frac{(1+\varsigma s)\rho+(1-\varsigma s)}{2} D_{-\frac{1+\varsigma s}{2}-i\rho}(sZ_b) D_{-\frac{1+\varsigma s}{2}+i\rho}(sZ_a) \chi_s \chi_{-s}^+ \right. \\ \left. - ie^{+i\frac{\pi}{4} \varsigma s} \frac{i\varsigma s p_y + m}{\sqrt{\mu B'}} D_{-\frac{1+\varsigma s}{2}-i\rho}(sZ_b) D_{-\frac{1-\varsigma s}{2}+i\rho}(sZ_a) \chi_s \chi_s^+ \right. \\ \left. + ie^{-i\frac{\pi}{4} \varsigma s} \frac{-i\varsigma s p_y + m}{\sqrt{\mu B'}} D_{-\frac{1-\varsigma s}{2}-i\rho}(sZ_b) D_{-\frac{1+\varsigma s}{2}+i\rho}(sZ_a) \chi_{-s} \chi_{-s}^+ \right\} e^{+i\frac{\pi}{4} \sigma_2} \quad (39)$$

where we have used the notations $Z_a = e^{i\frac{\pi}{4}}z_a$ and $Z_b = e^{-i\frac{\pi}{4}}z_b$. By factorizing we obtain

$$S^c(\mathbf{x}_b, \mathbf{x}_a) = i\varsigma \sum_{s=\pm 1} s \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} e^{-ip_0(t_b-t_a)+ip_y(y_b-y_a)} \frac{e^{-\frac{\pi}{2}\rho}}{2\sinh(\pi\rho)} e^{-i\frac{\pi}{4}\sigma_2} \left\{ \left[\frac{(1+s\varsigma)\frac{i\varsigma sp_y+m}{\sqrt{\mu B'}}+(1-s\varsigma)}{2} \right. \right. \\ \left. D_{-\frac{1+\varsigma s}{2}-i\rho}(sZ_b) \chi_s + i \frac{(1-s\varsigma)\frac{-i\varsigma sp_y+m}{\sqrt{\mu B'}}+(1+s\varsigma)}{2} e^{-i\frac{\pi}{4}s\varsigma} D_{-\frac{1-\varsigma s}{2}-i\rho}(sZ_b) \chi_{-s} \right] \left[\frac{(1+s\varsigma)\frac{-i\varsigma sp_y+m}{\sqrt{\mu B'}}+(1-s\varsigma)}{2} \right. \\ \left. D_{-\frac{1+\varsigma s}{2}+i\rho}(sZ_a) \chi_s^+ - i \frac{(1-s\varsigma)\frac{i\varsigma sp_y+m}{\sqrt{\mu B'}}+(1+s\varsigma)}{2} e^{+i\frac{\pi}{4}s\varsigma} D_{-\frac{1-\varsigma s}{2}+i\rho}(sZ_a) \chi_{-s}^+ \right] \left. \right\} \sigma_1 e^{+i\frac{\pi}{4}\sigma_2} \quad (40)$$

which can be further simplified to end up with

$$S_\varsigma^c(\mathbf{x}_b, \mathbf{x}_a) = i\varsigma \sum_{s=\pm 1} s \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} \Psi_{p_0 p_y}^{(\varsigma, s)}(x_b, y_b; t_b) \bar{\Psi}_{p_0 p_y}^{(\varsigma, s)}(x_a, y_a; t_a) \quad (41)$$

with $\bar{\Psi} = \Psi^\dagger \sigma_3$ and the normalized eigenspinors are given in compact form as

$$\Psi_{p_0 p_y}^{(\varsigma, s)}(x, y; t) = u_{p_0 p_y}^{(\varsigma, s)}(x, y; t) \chi_s + v_{p_0 p_y}^{(\varsigma, s)}(x, y; t) \chi_{-s} \quad (42)$$

where $u_{n,s}$ and $v_{n,s}$ are two-component defined by

$$u_{p_0 p_y}^{(\varsigma, s)}(x, y; t) = \frac{1}{2} \frac{e^{-\frac{\pi}{4}\rho}}{\sqrt{\sinh\left(\pi \frac{p_y^2+m^2}{\mu B'}\right)}} e^{-ip_0 t} e^{+ip_y y} \\ \left\{ \frac{(1+s\varsigma)\frac{i\varsigma sp_y+m}{\sqrt{\mu B'}}+(1-s\varsigma)}{2} D_{-\frac{1+\varsigma s}{2}-i\frac{p_y^2+m^2}{\mu B'}} \left(se^{-i\frac{\pi}{4}} \sqrt{\mu B'} \left(x - \frac{2\varsigma p_0 - \mu B}{\mu B'} \right) \right) \right. \\ \left. - i s \frac{(1-s\varsigma)\frac{-i\varsigma sp_y+m}{\sqrt{\mu B'}}+(1+s\varsigma)}{2} e^{-i\frac{\pi}{4}s\varsigma} D_{-\frac{1-\varsigma s}{2}-i\frac{p_y^2+m^2}{\mu B'}} \left(se^{-i\frac{\pi}{4}} \sqrt{\mu B'} \left(x - \frac{2\varsigma p_0 - \mu B}{\mu B'} \right) \right) \right\} \quad (43)$$

$$v_{p_0 p_y}^{(\varsigma, s)}(x, y; t) = \frac{1}{2} \frac{e^{-\frac{\pi}{4}\rho}}{\sqrt{\sinh\left(\pi \frac{p_y^2+m^2}{\mu B'}\right)}} e^{-ip_0 t} e^{+ip_y y} \\ \left\{ s \frac{(1+s\varsigma)\frac{i\varsigma sp_y+m}{\sqrt{\mu B'}}+(1-s\varsigma)}{2} D_{-\frac{1+\varsigma s}{2}-i\frac{p_y^2+m^2}{\mu B'}} \left(se^{-i\frac{\pi}{4}} \sqrt{\mu B'} \left(x - \frac{2\varsigma p_0 - \mu B}{\mu B'} \right) \right) \right. \\ \left. + i \frac{(1-s\varsigma)\frac{-i\varsigma sp_y+m}{\sqrt{\mu B'}}+(1+s\varsigma)}{2} e^{-i\frac{\pi}{4}s\varsigma} D_{-\frac{1-\varsigma s}{2}-i\frac{p_y^2+m^2}{\mu B'}} \left(se^{-i\frac{\pi}{4}} \sqrt{\mu B'} \left(x - \frac{2\varsigma p_0 - \mu B}{\mu B'} \right) \right) \right\}. \quad (44)$$

In matrix form, we can write the eigenspinors for $\varsigma = +1$

$$\Psi_{p_0 p_y}^{(+1, +1)}(x, y; t) = \frac{1}{2} \frac{e^{-\frac{\pi}{4}\rho}}{\sqrt{\sinh(\pi\rho)}} e^{-ip_0 t} e^{+ip_y y} \begin{pmatrix} \frac{ip_y+m}{\sqrt{\mu B'}} D_{-1-i\rho}(z_+) - ie^{-i\frac{\pi}{4}} D_{-i\rho}(z_+) \\ \frac{ip_y+m}{\sqrt{\mu B'}} D_{-1-i\rho}(z_+) + ie^{-i\frac{\pi}{4}} D_{-i\rho}(sz_+) \end{pmatrix} \quad (45)$$

$$\Psi_{p_0 p_y}^{(+1, -1)}(x, y; t) = \frac{1}{2} \frac{e^{-\frac{\pi}{4}\rho}}{\sqrt{\sinh(\pi\rho)}} e^{-ip_0 t} e^{+ip_y y} \begin{pmatrix} -D_{-i\rho}(-z_+) + i \frac{ip_y+m}{\sqrt{\mu B'}} e^{i\frac{\pi}{4}} D_{-1-i\rho}(-z_+) \\ D_{-i\rho}(-z_+) + i \frac{ip_y+m}{\sqrt{\mu B'}} e^{i\frac{\pi}{4}} D_{-\frac{1-\varsigma s}{2}-i\rho}(-z_+) \end{pmatrix} \quad (46)$$

and for $\varsigma = -1$

$$\Psi_{p_0 p_y}^{(-1, +1)}(x, y; t) = \frac{1}{2} \frac{e^{-\frac{\pi}{4}\rho}}{\sqrt{\sinh(\pi\rho)}} e^{-ip_0 t} e^{+ip_y y} \begin{pmatrix} D_{-i\rho}(z_-) - i \frac{ip_y+m}{\sqrt{\mu B'}} e^{i\frac{\pi}{4}} D_{-1-i\rho}(z_-) \\ D_{-i\rho}(z_-) + i \frac{ip_y+m}{\sqrt{\mu B'}} e^{i\frac{\pi}{4}} D_{-1-i\rho}(z_-) \end{pmatrix} \quad (47)$$

$$\Psi_{p_0 p_y}^{(-1, -1)}(x, y; t) = \frac{1}{2} \frac{e^{-\frac{\pi}{4}\rho}}{\sqrt{\sinh(\pi\rho)}} e^{-ip_0 t} e^{+ip_y y} \begin{pmatrix} -\frac{ip_y+m}{\sqrt{\mu B'}} D_{-1-i\rho}(-z_-) + ie^{-i\frac{\pi}{4}} D_{-i\rho}(-z_-) \\ \frac{ip_y+m}{\sqrt{\mu B'}} D_{-1-i\rho}(-z_-) + ie^{-i\frac{\pi}{4}} D_{-i\rho}(-z_-) \end{pmatrix} \quad (48)$$

with the change $z_\varsigma = e^{-i\frac{\pi}{4}}\sqrt{\mu B'}\left(x - \frac{2\varsigma p_0 - \mu B}{\mu B'}\right)$ and $\rho = \frac{p_y^2 + m^2}{\mu B'}$. These are exactly the wanted eigenspinors corresponding to the continuum energy spectrum $E = p_0$, which can be used to deal with different properties of the graphene systems [11] and related matters.

Now let us examine the asymptotic behavior of the obtained eigenspinors. Indeed, recalling the limiting cases when $z \rightarrow \infty$ [12]

$$D_\nu(z) \rightarrow z^\nu e^{-\frac{1}{4}z^2}, \quad |\arg z| \leq \frac{\pi}{2} \quad (49)$$

$$D_\nu(z) \rightarrow z^\nu e^{-\frac{1}{4}z^2} + \frac{\sqrt{2\pi}}{\Gamma(-\nu)} (-z)^{-\nu-1} e^{\frac{1}{4}z^2}, \quad \frac{\pi}{2} \leq |\arg z| \leq \pi, |\arg(-z)| \leq \frac{\pi}{2}. \quad (50)$$

These results can be used to show that the obtained eigenspinors reduce to the following two components for $\varsigma = +1$ and $x \rightarrow +\infty$

$$\Psi_{p_0, p_y}^{(+1, +1)}(x, y; t) \rightarrow \frac{1}{2} \frac{e^{-\frac{\pi}{2}\rho}}{\sqrt{\sinh(\pi\rho)}} e^{-ip_0 t} e^{+ip_y y} (\mu B')^{-1-i\frac{\rho}{2}} \left(x - \frac{2p_0 - \mu B}{\mu B'}\right)^{-1-i\rho} \quad (51)$$

$$\begin{aligned} & e^{\frac{i}{4}\mu B' \left(x - \frac{2p_0 - \mu B}{\mu B'}\right)^2} e^{i\frac{\pi}{4}} \begin{pmatrix} ip_y + m - \mu B'x + 2p_0 - \mu B \\ ip_y + m + \mu B'x - 2p_0 + \mu B \end{pmatrix} \\ \Psi_{p_0, p_y}^{(+1, -1)}(x, y; t) & \rightarrow \frac{1}{2} \frac{e^{\frac{\pi}{2}\rho}}{\sqrt{\sinh(\pi\rho)}} e^{-ip_0 t} e^{+ip_y y} (\mu B')^{-1-i\frac{\rho}{2}} \left(x - \frac{2p_0 - \mu B}{\mu B'}\right)^{-1-i\rho} \quad (52) \\ & e^{\frac{i}{4}\mu B' \left(x - \frac{2p_0 - \mu B}{\mu B'}\right)^2} \begin{pmatrix} -\mu B'x + 2p_0 - \mu B + ip_y + m \\ \mu B'x - 2p_0 + \mu B + ip_y + m \end{pmatrix} \\ & + \frac{1}{2} \frac{\sqrt{2\pi} e^{\frac{\pi}{2}\rho}}{\sqrt{\sinh(\pi\rho)}} e^{-ip_0 t} e^{+ip_y y} (\mu B')^{-\frac{1}{2}+i\frac{\rho}{2}} \left(x - \frac{2p_0 - \mu B}{\mu B'}\right)^{i\rho-1} \\ & e^{-\frac{i}{4}\mu B' \left(x - \frac{2p_0 - \mu B}{\mu B'}\right)^2} e^{i\frac{\pi}{4}} \begin{pmatrix} -\frac{1}{\Gamma(i\rho)} + i\frac{1}{\Gamma(1+i\rho)} (ip_y + m) \left(x - \frac{2p_0 - \mu B}{\mu B'}\right) \\ \frac{1}{\Gamma(i\rho)} + i\frac{1}{\Gamma(1+i\rho)} (ip_y + m) \left(x - \frac{2p_0 - \mu B}{\mu B'}\right) \end{pmatrix} \end{aligned}$$

and the remaining ones for $x \rightarrow -\infty$ and $\varsigma = -1$ for both limits can be worked out in the same manner. These summarize the most interesting results derived so far.

5 The probability of pair creation in vacuum in 2+1 dimensions

Recalling that the production rate of neutral fermions through the Pauli interaction in 3+1 dimensional was studied in [13]. Let us look for the probability of pair creation in vacuum in (2+1)-dimensions corresponding to our system. Indeed, we start with the relation gives the effective action [14]

$$\ln S_{\text{eff}}[x, y, t] = \text{Tr} \ln \left[\left(\gamma^\mu p_\mu - \frac{\mu}{2} \sigma^{\mu\nu} F_{\mu\nu} - m \right) \frac{1}{\gamma^\mu p_\mu - m} \right]. \quad (53)$$

Using the charge conjugation matrix C [15]

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T, \quad C\sigma^{\mu\nu} C^{-1} = -\sigma^{T\mu\nu} \quad (54)$$

to write (53) as

$$2 \ln S_{\text{eff}}[x, y, t] = \text{Tr} \ln \left[\left(\left(\gamma^\mu p_\mu - \frac{\mu}{2} \sigma^{\mu\nu} F_{\mu\nu} \right)^2 - m^2 \right) \frac{1}{p^2 - m^2} \right]. \quad (55)$$

By using the identity

$$\ln \frac{a}{b} = \int_0^{+\infty} \frac{d\lambda}{\lambda} \left(e^{i\frac{\lambda}{2}(b+i\epsilon)} - e^{i\frac{\lambda}{2}(a+i\epsilon)} \right) \quad (56)$$

we show that the probability, per unit time and per unit area (i.e. $\int dt dx dy = 1$), of neutral particle-antiparticle pair creation in the vacuum takes the form

$$\mathcal{W} = \text{Re} \int_0^\infty \frac{d\lambda}{\lambda} \text{Tr} \left[e^{i\frac{\lambda}{2}(\mathcal{H}+i\epsilon)} - e^{\frac{i\lambda}{2}(p_0^2 - p_x^2 - p_y^2 - m^2 + i\epsilon)} \right]. \quad (57)$$

Replacing the Hamiltonian \mathcal{H} (16) to obtain

$$\mathcal{W} = 2 \text{Re} \int_0^\infty \frac{d\lambda}{\lambda} e^{-\frac{i\lambda}{2}m^2} \text{Tr} \left[\int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \frac{dp_y}{2\pi} e^{-\frac{i\lambda}{2}p_y^2} K^{os}(x, x; \lambda) \cosh\left(\frac{\lambda\mu B'}{4}\right) - \frac{\pi}{(2\pi)^3} \frac{\lambda}{2} \sqrt{\frac{\pi}{\frac{i\lambda}{2}}} \right] \quad (58)$$

where

$$K^{os}(x, x; \lambda) = \sqrt{\frac{\frac{\mu B'}{2}}{2i\pi \sinh\left(\frac{\lambda\mu B'}{2}\right)}} \exp\left(i\frac{2}{\mu B'} \left(p_0 - \varsigma\mu \frac{B+B'x}{2}\right)^2 \tanh\left(\frac{\lambda\mu B'}{4}\right)\right). \quad (59)$$

Then by performing the integration over p_0 and p_y to obtain

$$\sqrt{\frac{\frac{\mu B'}{2}}{2i\pi \sinh\left(\frac{\lambda\mu B'}{2}\right)}} \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} e^{i\frac{2}{\mu B'} \left(p_0 - \varsigma\mu \frac{B+B'x}{2}\right)^2 \tanh\left(\frac{\lambda\mu B'}{4}\right)} = \frac{1}{2\pi} \frac{\frac{\mu B'}{2}}{2 \sinh\left(\frac{\lambda\mu B'}{4}\right)} \quad (60)$$

and

$$\int_{-\infty}^{+\infty} \frac{dp_y}{2\pi} e^{-\frac{i\lambda}{2}p_y^2} = \frac{1}{2\pi} \sqrt{\frac{\pi}{\frac{i\lambda}{2}}}. \quad (61)$$

Combining all to write

$$\mathcal{W} = 2 \text{Re} \int_0^\infty \frac{d\lambda}{\lambda} e^{-\frac{i\lambda}{2}(m^2 - i\epsilon)} \left(\frac{\mu B'}{4(2\pi)^2} \sqrt{\frac{\pi}{\frac{i\lambda}{2}}} \coth\left(\frac{\lambda\mu B'}{4}\right) - \frac{\pi}{(2\pi)^3} \frac{\lambda}{2} \sqrt{\frac{\pi}{\frac{i\lambda}{2}}} \right) \quad (62)$$

or equivalently

$$\begin{aligned} \mathcal{W} &= \frac{1}{8\pi^2} \sqrt{\frac{2\pi}{i}} \int_0^\infty \frac{d\lambda}{\lambda^{\frac{3}{2}}} e^{-\frac{i\lambda}{2}(m^2 - i\epsilon)} \left(\frac{\mu B'}{4} \coth\left(\frac{\lambda\mu B'}{4}\right) - \frac{1}{\lambda} \right) \\ &\quad + \frac{1}{8\pi^2} \sqrt{\frac{2\pi}{-i}} \int_0^\infty \frac{d\lambda}{\lambda^{\frac{3}{2}}} e^{\frac{i\lambda}{2}(m^2 + i\epsilon)} \left(\frac{\mu B'}{4} \coth\left(\frac{\lambda\mu B'}{4}\right) - \frac{1}{\lambda} \right). \end{aligned} \quad (63)$$

By making the change of variable $\lambda \rightarrow \lambda' = e^{i\pi} \lambda$ in the second line of the above expression and after some steps we get

$$\mathcal{W} = \frac{1}{8\pi^2} \sqrt{\frac{2\pi}{i}} \int_{-\infty}^\infty \frac{d\lambda}{\lambda^{\frac{3}{2}}} e^{-\frac{i\lambda}{2}(m^2 + i\epsilon)} \left(\frac{\mu B'}{4} \coth\left(\frac{\lambda\mu B'}{4}\right) - \frac{1}{\lambda} \right). \quad (64)$$

Using the residue theorem for the simple poles $\lambda = \pm i \frac{4n\pi}{\mu B'}$, to obtain

$$\mathcal{W} = \frac{1}{4\sqrt{\pi}} \sum_{n=1}^{+\infty} \frac{1}{\left(\frac{4\pi n}{\mu B'}\right)^{\frac{3}{2}}} e^{-\frac{2n\pi m^2}{\mu B'}}. \quad (65)$$

At this level we have some comments in order. Firstly, (65) is similar to the result obtained in dealing with the case of an electric field [16]. Secondly, we observe that the neutral particle creation probability is an exponentially decreasing function with respect to the inverse of the $\mu B'$. This result is similar to the Schwinger process of electron-positron pair creation in the strong electric field [6].

6 Conclusion

We have considered (2+1)-dimensional Dirac equation related to a relativistic half spin particle in interaction with inhomogeneous magnetic field. Using the global projection as well as the path integral formalism to write down the corresponding propagator as causal Green function in terms of different physical quantities. These allowed to end up with an interesting relation between the matrix elements of both projections $S^c(\mathbf{x}_b, \mathbf{x}_a)$ and $S_g^c(\mathbf{x}_b, \mathbf{x}_a)$ (11). To go further, we have fixed the gauge field in terms of two different magnetic fields and therefore explicitly got the corresponding Hamiltonian. This is used together with the boundary conditions and the propagator associated to the harmonic oscillator to map the appropriate Green function in terms of the parabolic cylindrical functions.

Later on, we have used the differential equation of the Green function together with some relevant properties of the parabolic cylindrical functions to further simplify the matrix elements of the Pauli-Dirac operator. We have made several algebras based on the residue calculus to finally end up with the eigenspinors in terms of the physical parameters. Their asymptotic behaviors were also examined for the limiting case $x \rightarrow +\infty$ and the value $\varsigma = +1$. Subsequently, we have used the effective action corresponding to our system to study the neutral particle creation probability. Indeed, this allowed to obtain a result depending on the included field B' and the additional spin magnetic moment μ .

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